

**We Claim:**

1. A method of detecting one or more patterns embedded in an image, each pattern having been formed substantially from a one-dimensional basis function, said method  
5 comprising the steps of:

(a) calculating a projective transform of said image;

(b) calculating a 1-D correlation between the projective transform and said basis function for a selection of angles; and

(c) finding one or more peaks of said correlation, wherein the position of each  
10 of said peaks provides spatial parameters of one of said one or more embedded patterns.

2. A method of determining transformations applied to an image, said image having at least three patterns embedded therein, each pattern having been formed substantially from a one-dimensional basis function being substantially scale invariant as herein defined, said  
15 method comprising the steps of:

(a) calculating a projective transform of said image;

(b) calculating a 1-D correlation between the projective transform and said basis function for a selection of angles;

(c) finding peaks of said correlation, wherein the positions of said peaks  
20 provide spatial parameters of said embedded patterns; and

(d) determining from said spatial parameters said transformations.

3. A method as claimed in claim 1, wherein step (a) comprises the sub-steps of:

(a1) transforming said image to the frequency domain using a two-dimensional  
25 transform;

(a2) resampling the transformed image onto a quasi-polar map; and

(a3) inverse transforming the resampled image to the space domain using a one-dimensional transform.

4. A method as claimed in claim 1, wherein step (b) comprises the sub-steps of:

5 (b1) transforming said projective transform to the frequency domain using a one-dimensional transform;

(b2) transforming said basis function to the frequency domain using a one-dimensional transform;

10 (b3) multiplying the transformed projective transform with the transformed basis function along radial lines to create a result; and

(b4) inverse transforming said result to the space domain using a one-dimensional transform.

5. A method as claimed in claim 1, wherein step (b) comprises the sub-steps of:

15 (b1) determining a scale factor ;

(b2) scaling said basis function with said scale factor to form a scaled basis function; and

(b3) cross-correlating said scaled basis function and said projective transform for a selection of angles.

20

6. A method as claimed in claim 5, wherein step (b1) comprises the sub-steps of:

(b1i) calculating a magnitude of the 1-dimensional Fourier transform of said basis function ;

25 (b1ii) calculating a magnitude of the 1-dimensional Fourier transform of said projective transform;

(b1iii) resampling said magnitudes onto a logarithmic sampling space;

(b1iv) correlating the resampled magnitudes of the 1-dimensional Fourier transforms of said basis function and projective transform along constant radial lines; and

(b1v) finding at least one absolute peak in the result of step (b1iii), wherein the location of said peak is used to determine said scale factor.

5

7. A method of detecting one or more patterns embedded in an image, each pattern having been formed substantially from a one-dimensional basis function, said method comprising the steps of:

transforming said image to the frequency domain using a two-dimensional  
10 transform;

resampling the transformed image onto a quasi-polar map;

transforming said basis function to the frequency domain using a one-dimensional transform;

15 multiplying the resampled transformed image with the transform of said basis function along radial lines to create a first result;

inverse transforming said first result to the space domain using a one-dimensional transform to create a second result; and

finding one or more peaks of said second result, wherein the position of each of said peaks provides spatial parameters of one of said one or more embedded patterns.

20

8. A method of determining transformations applied to a first image, said first image being a transformed copy of a second image, said second image having at least three patterns embedded therein, each pattern having predefined parameters and having been formed substantially from a one-dimensional basis function being substantially scale invariant as  
25 herein defined, said method comprising the steps of:

calculating a projective transform of said first image;

calculating a 1-D correlation between the projective transform and said basis function for a selection of angles;

finding peaks of said correlation, each peak corresponding with one of said embedded patterns; and

5 determining said transformations from the positions of said peaks and said parameters of said patterns embedded into said second image.

9. A method of embedding a watermark into an image, said method comprising the steps of:

10 maintaining a basis function, said basis function being a substantially scale invariant one-dimensional function excluding the function  $f(r) = \cos(\alpha \log|r| + c)$  wherein  $\alpha$  and  $c$  are constants;

forming one or more patterns from said basis function, each pattern having no variation in one direction; and

15 adding said basis pattern(s) to said image.

10. A method as claimed in claim 9, wherein said scale invariant one-dimensional function is a function wherein the profile of the phase of said function is substantially invariant after scaling within a predefined range of values.

20

11. A method as claimed in claim 10, wherein said scale invariant one-dimensional function is selected from the group comprising:

$$\chi(r) = r^p \exp\left(i \frac{\alpha}{\gamma} r^\gamma\right);$$

$$q_1(r) = [\text{sgn}(r)]^k \exp(i(\alpha r^{-p} + \varepsilon));$$

$$q_2(r) = \sum_{n=N_1}^{N_2} \xi_n q_0(|\mu|^n r); \text{ and}$$

$$q_3(r) = [\text{sgn}(r)]^k r^p \exp\left(i \frac{\alpha}{\gamma} r^\gamma\right); \text{ wherein } \gamma, \varepsilon, p, N_1, N_2, \xi_n, \alpha, k \text{ and } \mu \text{ are constants}$$

and  $q_0$  is an arbitrary function.

5 12. A method as claimed in claim 9, wherein said basis function is a complex function and said one or more patterns are real valued.

13. A method as claimed in claim 9, wherein each pattern is added with a respective displacement from the image centre and a respective orientation.

10

14. An apparatus for detecting one or more patterns embedded in an image, each pattern having been formed substantially from a one-dimensional basis function, said apparatus comprising:

means for calculating a projective transform of said image;

15 means for calculating a 1-D correlation between the projective transform and said basis function for a selection of angles; and

means for finding one or more peaks of said correlation, wherein the position of each of said peaks provides spatial parameters of one of said one or more embedded patterns.

20 15. An apparatus for determining transformations applied to an image, said image having at least three patterns embedded therein, each pattern having been formed substantially from a one-dimensional basis function being substantially scale invariant as herein defined, said apparatus comprising:

means for calculating a projective transform of said image;

means for calculating a 1-D correlation between the projective transform and said basis function for a selection of angles;

means for finding peaks of said correlation, wherein the position of said peaks provide spatial parameters of said embedded patterns; and

5 means for determining from said spatial parameters said transformations.

16. An apparatus for detecting one or more patterns embedded in an image, each pattern having been formed substantially from a one-dimensional basis function, said apparatus comprising:

10 means for transforming said image to the frequency domain using a two-dimensional transform;

means for resampling the transformed image onto a quasi-polar map;

means for transforming said basis function to the frequency domain using a one-dimensional transform;

15 means for multiplying the resampled transformed image with the transform of said basis function along radial lines to create a first result;

means for inverse transforming said first result to the space domain using a one-dimensional transform to create a second result; and

20 means for finding a peak of said second result, wherein the position of each of said peaks provides spatial parameters of one of said one or more embedded patterns.

17. An apparatus for determining transformations applied to a first image, said first image being a transformed copy of a second image, said second image having at least three patterns embedded therein, each pattern having predefined parameters and having been  
25 formed substantially from a one-dimensional basis function being substantially scale invariant as herein defined, said apparatus comprising:

means for calculating a projective transform of said first image;

means for calculating a 1-D correlation between the projective transform and said basis function for a selection of angles;

means for finding peaks of said correlation, each peak corresponding with one of said embedded patterns; and

means for determining said transformations from the positions of said peaks and said parameters of said patterns embedded into said second image.

18. An apparatus for embedding a watermark into an image, said apparatus comprising:

means for maintaining a basis function, said basis function being a substantially scale invariant one-dimensional function excluding the function  $f(r) = \cos(\alpha \log|r| + c)$  wherein  $\alpha$  and  $c$  are constants;

means for forming one or more patterns from said basis function, each pattern having no variation in one direction; and

means for adding said basis pattern(s) to said image.

19. A program stored in a memory for detecting one or more patterns embedded in an image, each pattern having been formed substantially from a one-dimensional basis function, said program comprising:

code for calculating a projective transform of said image;

code for calculating a 1-D correlation between the projective transform and said basis function for a selection of angles; and

code for finding one or more peaks of said correlation, wherein the position of each of said peaks provides spatial parameters of one of said one or more embedded patterns.

20. A program stored in a memory for determining transformations applied to an image, said image having at least three patterns embedded therein, each pattern having been formed substantially from a one-dimensional basis function being substantially scale invariant as herein defined, said program comprising:

5 code for calculating a projective transform of said image;

code for calculating a 1-D correlation between the projective transform and said basis function for a selection of angles;

code for finding peaks of said correlation, wherein the positions of said peaks provide spatial parameters of said embedded patterns; and

10 code for determining from said spatial parameters said transformations.

21. A program stored in a memory for detecting one or more patterns embedded in an image, each pattern having been formed substantially from a one-dimensional basis function, said program comprising:

15 code for transforming said image to the frequency domain using a two-dimensional transform;

code for resampling the transformed image onto a quasi-polar map;

code for transforming said basis function to the frequency domain using a one-dimensional transform;

20 code for multiplying the resampled transformed image with the transform of said basis function along radial lines to create a first result;

code for inverse transforming said first result to the space domain using a one-dimensional transform to create a second result; and

25 code for finding one or more peaks of said second result, wherein the position of each of said peaks provides spatial parameters of one of said one or more embedded patterns.



22. A program stored in a memory for determining transformations applied to a first image, said first image being a transformed copy of a second image, said second image having at least three patterns embedded therein, each pattern having predefined parameters and having been formed substantially from a one-dimensional basis function being substantially scale invariant as herein defined, said program comprising:

code for calculating a projective transform of said first image;

code for calculating a 1-D correlation between the projective transform and said basis function for a selection of angles;

code for finding peaks of said correlation, each peak corresponding with one of said embedded patterns; and

code for determining said transformations from the positions of said peaks and said parameters of said patterns embedded into said second image.

23. A program stored in a memory for embedding a watermark into an image, said program comprising:

code for maintaining a basis function, said basis function being a substantially scale invariant one-dimensional function excluding the function  $f(r) = \cos(\alpha \log|r| + c)$  wherein  $\alpha$  and  $c$  are constants;

code for forming one or more patterns from said basis function, each pattern having no variation in one direction; and

code for adding said basis pattern(s) to said image.